

Annihilation explosions in macroscopic polyelectrons. Photon detonation.

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Abstract

Annihilation of the electron-positron pairs in macroscopic polyelectrons is considered. It is shown that very fast collapse of the spatial area occupied by macroscopic polyelectron (or dense electron-positron plasma) produces an instant annihilation of a very large number of electron-positron pairs. This phenomenon corresponds to the so-called annihilation explosion. Annihilation of each electron-positron pair is a highly exothermic process. Therefore, in dense electron-positron plasma one can observe a very interesting phenomenon of photon detonation, i.e. a self-organized formation and propagation of the detonation wave which coincides with the annihilation wave. The photon detonation can be used in many applications, including many military and astrophysical problems.

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I. INTRODUCTION

In our previous work [1] we have considered annihilation of the electron-positron pairs in the three-body Ps^- ($= e^-e^+e^- = e_2^-e^+$) ion and four-body bi-positronium ‘molecule’ Ps_2 ($e^-e^+e^-e^+ = e_2^-e_2^+$). The main result obtained in [1] indicates clearly that annihilation of the electron-positron pairs in these two systems proceeds mainly with the emission of the two and three photons. Briefly, this means that the annihilation equations in the Ps^- ion and Ps_2 ‘molecule’ can be written in one of the two following forms

$$e^- + e^+ = \hbar\omega_1 + \hbar\omega_2 \quad , \quad \text{or} \quad e^- + e^+ = \hbar\omega_1 + \hbar\omega_2 + \hbar\omega_3 \quad (1)$$

Other annihilation channels, e.g., four-photon and five-photon annihilations, can be ignored in the first approximation which appears to be remarkably accurate. Moreover, these results for the Ps^- ion and Ps_2 system can be generalized to arbitrary polyelectrons, including macroscopic polyelectrons. In other words, for an arbitrary polyelectron we can also consider only the two- and three-photon annihilation of the electron-positron pairs. This allows one to study the electron-positron plasmas with possible annihilation of particles, since the system of governing equations is now written in the closed form. The approximate analytical and accurate numerical solutions of this system of equations can be found by applying a number of well-known procedures (see, e.g., [2], [3]).

In general, the solution of the governing equations for electron-positron plasma depends upon the initial and boundary conditions which exist in reality. It is clear *a priori* that electron-positron annihilation is accelerated when the density of electron-positron plasma increases. At very large densities of such plasma annihilation of the electron-positron pairs proceeds instantly. This phenomenon is called the photon explosion (or annihilation explosion). At relatively large densities of the electron-positron plasma one can observe a very interesting phenomenon of photon detonation. In this study by photon detonation we mean a self-organized formation and propagation of the detonation wave in the dense macroscopic polyelectrons. In this detonation wave the density of the electron-positron plasma increases suddenly to very large values. This substantially accelerates annihilation of the electron-positron pairs. In other words, the detonation wave in dense electron-positron plasmas always propagates as an annihilation wave.

Our main goal below is to consider annihilation of the (e^-, e^+) –pairs in macroscopic polyelectrons. In this work the macroscopic polyelectron (or polylepton) designates the elec-

trically neutral (or quasi-neutral) macroscopic system which contains approximately equal (and very large) numbers of electrons and positrons. In general, an arbitrary macroscopic polyelectron can be designated as $e_n^- e_m^+$, where $n \approx m \approx N_A$ and N_A is the Avogadro number. Below, in this study all macroscopic polyelectrons are assumed to be neutral, i.e. $m = n$, or quasi-neutral, i.e. $m \approx n$ and $|m - n| \ll \min(m, n)$. Macroscopic polyelectrons have a large number of unique properties since such systems represent a different, non-Born-Oppenheimer world. In this study we restrict ourselves only to the analysis of annihilation processes in macroscopic polyelectrons. It should be emphasized that boundness of macroscopic polyelectrons is not crucial for the goals of this study. Below, we shall assume that a large number of electrons and positrons is somehow confined in one spatial area. There is a number of approaches, e.g., radiative or magnetic ablation, which can be used to achieve such a confinement in reality. Note also that our definition of polyelectrons also includes the case of the confined electron-positron plasma.

II. PHOTON DETONATION IN MACROSCOPIC POLYELECTRONS.

In general, annihilation of the electron-positron pairs is a highly exothermic process. In fact, the energy E_a released during annihilation of one (e^-, e^+) -pair is $E_a \geq 2m_e c^2 \approx 1.022 \text{ MeV}$. It follows from here that the energy released per unit mass of the annihilating electron-positron mixture is quite comparable with the analogous energy released during nuclear fission and/or thermonuclear burning of the 1:1 deuterium-tritium mixture (below, DT-mixture). An accurate evaluation shows that the total annihilation of one gram of the electron-positron (1:1) mixture produces the energy $\approx 4.93 \cdot 10^{10} \text{ J}$. The same amount of energy can be obtained from thermal explosion of 11.8 tonnes of TNT. An analogous amount of thermal energy released during the complete fission of all nuclei from one gram of Pu-239 is ≈ 16.4 tonnes TNT. Thermonuclear burning of one gram of the 1:1 deuterium-tritium mixture releases at least 13.8 tonnes TNT.

Bearing this in mind and by applying the well known Khariton's theorem (see, e.g., [4] and references therein) one finds that all macroscopic polyelectrons are able to detonate in respect to the annihilation, if their spatial radii exceed some minimal (or critical) value. In a slightly more general form, we can say that detonation can be achieved, if the product x of spatial radius and electron-positron density, i.e. $x = \rho r$, is relatively large. In special literature,

the value $x = \rho r$ is called the burn-up parameter. Therefore, the detonation criterion can also be formulated in the form $x \geq x_{cr}$, where x_{cr} is the critical value of the burn-up parameter for the electron-positron plasma with density ρ_0 . In reality, the minimal spatial radius can be extremely large, e.g., a few dozens of kilometers. Note that there is a principal difference between the annihilation explosion (or photon explosion) and photon detonation. Annihilation explosion can always be observed, e.g., if the density of annihilating matter is increased to the infinity in a ‘very short’ time t_e . In other words, the photon explosion will proceed in such cases, when the density of electron-positron macroscopic mixture (i.e. polyelectron) increases to the infinity with time t as

$$\rho(t) = \frac{A}{(t - t_e)^n} \quad (2)$$

where A and n are some positive constants (in all actual cases $n > 1$). The explosion time t_e is assumed to be short in comparison with the life-time of the electron-positron mixture. It should be mentioned that in contrast with the photon (or annihilation) explosion, the photon detonation is a self-organized motion which may arise in any annihilating matter, e.g., in the macroscopic electron-positron mixture. In fact, the regions of high density of the electron-positron plasma can also be created during such a motion. Annihilation of electron-positron pairs in such high-dense areas proceeds instantly.

Below, we restrict ourselves to the consideration of the photon detonation only, since this phenomenon is of great interest in numerous applications as well as for the future theoretical development. A general theory of photon explosions will be discussed elsewhere. In general, the photon detonation wave propagating in dense electron-positron mixtures must be a very intense source of X -ray radiation. Note also that the annihilation of the (e^-, e^+) -pairs in macroscopic volumes does not require any minimal critical density and/or threshold temperature for its ignition. On the other hand, the rate of energy release in any system undergoing annihilation rapidly increases at high compressions, since the reaction probability is proportional to the expectation value of electron-positron delta-function $\langle \delta_{+-} \rangle$. The proportionality of the overall reaction rate to the expectation value of the two-particle delta-function is also true for all working systems based on nuclear fusion. It is also clear that at high densities the photons emitted during the annihilation of the (e^-, e^+) -pair have significantly better probabilities to redeposit their energy into surrounding electrons, positrons and atoms. The condition for ‘sufficient redeposition’ of energy allows one to

derive the rigorous criterion of ignition (or ignition criterion).

Formally, the ignition criterion (or photon detonation criterion) can be written in the form $L \geq \lambda_R$, where λ_R is the Rosseland mean free path of the annihilation photon and L is the minimal spatial dimension of the electron-positron mixture (or plasma). The physical meaning of this condition ($L \geq \lambda_R$) is very transparent: annihilation photons must leave a substantial part of their energy inside of the igniting electron-positron mixture. In fact, such an ignition criterion is only a necessary condition. For actual ignition it must be $L \gg \lambda_R$. By introducing the Rosseland mean opacity $R_0 = \frac{1}{\lambda_R \rho}$, where ρ is the macroscopic density of the electron-positron mixture, one can re-write the ignition criterion in the form $L \gg \frac{1}{\rho R_0}$. In general, the Rosseland mean opacity rapidly increases with the density of the plasma. At small densities $\rho \leq 1 \cdot 10^{-3} \text{ g} \cdot \text{cm}^{-3}$ the numerical values of R_0 are very small. Therefore, to decrease the critical size of the annihilating electron-positron mixture one needs to compress this mixture to very large densities ρ .

Let us write the equations of motion and energy equations for the electron-positron plasma which also includes radiation transfer. At this stage of our analysis we shall ignore any presence of atoms, ions and bare nuclei. Below, the notations ρ_- and ρ_+ stand for the densities of electrons and positrons, respectively. The total density of polyelectron $\rho_- + \rho_+$ is designated by ρ . The value defined by the equation $v = \frac{1}{\rho}$ is the specific volume. To simplify the following analysis we restrict ourselves to the case of planar geometry. In planar geometry the elementary mass of the macroscopic polyelectron (or electron-positron plasma) is $dm = \rho dx$. The velocity v of the moving part of polyelectron is $u = \frac{\partial x}{\partial t}$. In this notation the equation of motion takes the form [5]

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho} \frac{\partial(P+q)}{\partial x} = -\frac{\partial(P+q)}{\partial m} \quad (3)$$

where $P = P_e + P_r = P_- + P_+ + P_r$ is the total pressure and q is an additional pressure related with the artificial viscosity (Von-Neuman artificial viscosity). The notation P_r is the radiation pressure. The electron (P_-) and positron (P_+) pressures are easily related to the corresponding temperatures, since

$$P_- = k \left(\frac{\rho_-}{m_e} \right) T_- \quad , \quad P_+ = k \left(\frac{\rho_+}{m_e} \right) T_+ \quad (4)$$

where k is the Boltzmann constant and m_e is the electron mass. Below, we shall assume that $T_- = T_+ = T_e$. Also by assuming that all photons achieve the Planckian equilibrium with

the temperature T_r one finds the following expression for the radiation pressure $P_r = \frac{4\sigma}{3c}T_r^4$, where σ is Stephan-Boltzmann constant and c is the speed of light. In general, the electron and radiation temperatures differ from each other, i.e. $T_e \neq T_r$.

Let us describe the following two-temperature code which is based on the radiation diffusion model. In the case of planar geometry the coupled energy equations take the form

$$\frac{\partial T_e}{\partial t} = \frac{1}{C_{ve}} \left[v \frac{\partial}{\partial x} \left(K_e \frac{\partial T_e}{\partial x} \right) - A_{er}(T_e - T_r) + \frac{dS_e}{dt} - \left(P_e + C_{ve} \frac{\partial T_e}{\partial v} \right) \frac{\partial v}{\partial t} \right] \quad (5)$$

$$\frac{\partial T_r}{\partial t} = \frac{1}{C_{vr}} \left[v \frac{\partial}{\partial x} \left(K_r \frac{\partial T_r}{\partial x} \right) + A_{er}(T_e - T_r) - \left(P_r + C_{vr} \frac{\partial T_r}{\partial v} + \frac{4\sigma}{c} T_r^4 \right) \frac{\partial v}{\partial t} \right], \quad (6)$$

where $C_{ve} = \frac{3}{2} \frac{k}{m_e}$ and $C_{vr} = \frac{16\sigma}{c\rho} T_r^3$ are the specific heats for electron-positron plasma and radiation, respectively. Also in these equations the corresponding electron/positron and radiation conductivities K_e and K_r are

$$K_e = 1.9369 \left(\frac{2}{\pi} \right)^{\frac{3}{2}} \frac{k^{\frac{7}{2}} T^{\frac{5}{2}}}{m_e^{\frac{1}{2}} e^4} \approx 0.98385 \cdot \frac{k^{\frac{7}{2}} T^{\frac{5}{2}}}{m_e^{\frac{1}{2}} e^4}, \quad K_r = \frac{16\sigma \lambda_R T_r^3}{3} \quad (7)$$

where λ_R is the Rosseland mean free path mentioned above. The electron/positron-radiation coupling constant A_{er} is represented in the form $A_{er} = C_{ve}(\nu_b + \nu_C)$, where C_{ve} is the specific heat of the electron-positron plasma, while ν_b and ν_C are the overall bremsstrahlung rate and Compton scattering rate, respectively. In our radiation diffusion model the Compton scattering rate ν_C can be evaluated from the following formula

$$\nu_C = 134.04 \cdot \frac{\sigma R e^2}{(m_e c^2)^2} \xi T_r^4 \quad (8)$$

where $R = kN_A$ is the universal gas constant and ξ is the numerical factor ($\xi \approx 2$). To obtain the exact value of ξ one needs to take into account an obvious similarity between hydrogen atom H and two-body positronium system Ps ($= e^-e^+$). Such a similarity can be found for the bound state spectra of these systems, their continuous spectra, photodetachment cross-sections, etc. In fact, it can be traced even for corresponding negatively charged ions H^- and Ps^- each of which has only one stable bound state (1^1S -state [6]). This means that for electron-positron plasma we can use the known formula for the Compton scattering rate in the pure hydrogen (see, e.g., [5]). The only change in this formula is related to the fact that the Ps (e^+e^-) system has spatial radius which is twice larger than the analogous spatial radius of the hydrogen atom H. The same approach can be used to evaluate a large number of other properties, e.g., the overall bremsstrahlung rate and Von-Neuman artificial viscosity.

The overall bremsstrahlung rate in macroscopic polyelectrons is written in the form

$$\nu_b = \frac{64\sqrt{2}\pi}{3} \cdot \frac{\alpha e^2 R^2 \rho \xi^\beta}{k\sqrt{m_e kT}} \cdot G\left(\frac{T_r}{T_e}\right) \approx 0.390224 \cdot \frac{e^2 R^2 \rho \xi^\beta}{k\sqrt{m_e kT}} \cdot G\left(\frac{T_r}{T_e}\right) \quad (9)$$

where $\alpha = \frac{e^2}{2\pi\hbar c}$ is the fine structure constant and $G(x)$ is the so-called universal bremsstrahlung function

$$G(y) = \frac{1}{y-1} \int_0^\infty \frac{dx f(x) \left\{ 1 - \exp\left[\frac{x(y-1)}{y}\right] \right\}}{1 - \exp\left(-\frac{x}{y}\right)} \quad (10)$$

where the function $f(y)$ represents the bremsstrahlung emission spectrum. Its explicit form is

$$f(y) = \int_1^\infty \ln(\sqrt{z} + \sqrt{z+1}) \exp(-yz) dz = \exp(-y) \times \quad (11)$$

$$\int_0^\infty \ln(\sqrt{z+1} + \sqrt{z+2}) \exp(-yz) dz = \frac{\exp(-\frac{5}{2}y)}{2y} I_0\left(\frac{y}{2}\right) - \frac{1 + \sqrt{2}}{y} \exp(-y)$$

where $I_0(x)$ is the modified Bessel function (see, e.g., [7]). Note that, if $T_r \gg T_e$, then $\frac{T_r}{T_e} \rightarrow 1$, $G(x) \rightarrow 1$ and ν_b is the pure bremsstrahlung rate. In the opposite case, i.e. when $T_r \gg T_e$, then $\frac{T_r}{T_e} \rightarrow \infty$, $G(x) \rightarrow \frac{\pi^2}{4}$ and ν_b is the rate of inverse bremsstrahlung [5].

The equations Eq.(3), Eq.(5) and Eq.(6) govern the time-evolution of macroscopic polyelectrons. Such a time-evolution also includes annihilation of the electron-positron pairs in polyelectrons. Our main hypothesis is related to the fact that at some conditions the propagation of electron-positron annihilation will be described as a propagation of moving surfaces in dense polyelectrons. In these cases we can observe the photon detonation. Currently, we cannot prove that such a detonation can be detected at realistic densities of the compressed polyelectrons $\rho \leq 1 \text{ g} \cdot \text{cm}^{-3}$. At very large densities, e.g., $\rho \approx 8.5 \cdot 10^2 \text{ g} \cdot \text{cm}^{-3}$, we have, in fact, a sudden X -ray flush of annihilation γ -quanta, i.e. the annihilation explosion at high densities. In this case any thermal equilibrium between particles and radiation cannot be reached. With our equilibrium code described above, it is hard to understand all details of the transition to the non-equilibrium process. At this moment our non-equilibrium code written for electron-positron plasmas does not work due to some unsolved computational problems. In addition to this, such high densities in polyelectrons ($\rho \geq 1 \cdot 10^3 \text{ g} \cdot \text{cm}^{-3}$) are comparable with the corresponding Fermi limit (see below). Polyelectrons compressed to such densities must be considered as degenerated Fermi systems. This drastically complicates annihilation analysis in highly compressed polyelectrons. Nevertheless, some predictions can be made in such cases too, and we hope to report the results of our analysis in future works.

III. THE LOW-TEMPERATURE FERMI LIMIT. STATISTICS OF PARTICLES IN POLYELECTRONS.

Let us briefly describe the Fermi degeneration of identical fermions (electrons and/or positrons) at high densities and relatively low temperatures. Such a degeneration is crucial for evaluation of the feasibility of various explosive devices based on extremely high compressions (see, e.g., [5]). As is well known (see e.g. [9]), if the density of the cold matter is quite high, then all electrons should be considered as degenerated fermions. In this case the internal electron pressure P_e as well as the speed of sound $S_s \approx \sqrt{P_e}$ in a cold electron plasma can suddenly increase to extremely large values. In highly-compressed polyelectrons we always have both electrons and positrons; and the overall pressure related to the electron/positron degeneracy reaches very large values. The appropriate disassembly time, $\tau \approx \frac{R}{S_s}$, becomes very short and annihilation in these polyelectrons cannot proceed effectively. Here R means the spatial radius of the highly compressed volume in the electron-positron plasma, while S_s is the speed of sound. In fact, the correction on the electron/positron degeneracy is needed when the real temperature T is comparable with the so-called equivalent Fermi temperature T_{ef} [9]. For the equimolar electron-positron plasmas the explicit expression for the equivalent Fermi temperature T_{ef} (in keV) takes the form

$$T_{ef}(keV) = \frac{2}{5} \left(\frac{3N_A M_p}{8\pi m_e} \right)^{\frac{2}{3}} \cdot \frac{2\pi^2 \hbar^2}{m_e} \rho_0^{\frac{2}{3}} \approx 1.39481 \cdot \rho_0^{\frac{2}{3}} \quad (12)$$

where N_A is the Avogadro number, \hbar is the Planck constant, M_p is the proton mass and m_e electron mass, respectively [8]. Also, in this equation ρ_0 is the macroscopic density of the electron-positron plasma in $g \cdot cm^{-3}$. As follows from Eq.(12) for $T = 0.1 \text{ keV}$ the pressure of the degenerated electron-positron mixture will be ≈ 14 times larger than its pressure evaluated with the use of formulas for an ideal gas.

Note also that the actual statistics of particles in macroscopic polyelectrons and/or in highly compressed electron-positron mixtures is still an open question. Based on ‘atomic analogies’ one can expect that both electrons and positrons must be considered as the two different types of particles. But, this picture is realistic only for very small, non-relativistic energies. Moreover, applications of these ‘atomic analogies’ to real polyelectrons often produce various contradictions with reality even for non-relativistic polyelectrons. The common reason of such contradiction is clear; since all electrons and positrons in polyelectrons must

be considered as the two states of the same particle [11]. This generates a special ‘exchange’ interaction between particles in polyelectrons. Due to such an exchange interaction all polyelectrons with equal (and even) numbers of positrons and electrons have remarkable stability. For instance, the ground 1S –state in the bi-positronium Ps_2 is very stable. Moreover, all properties of this state are invariant under charge conjugation \mathcal{C} , i.e., when $e \leftrightarrow -e$. The bi-positronium ion Ps_2e^- is not invariant under charge conjugation, since $\mathcal{C}(\text{Ps}_2e^-) = \text{Ps}_2e^+ \neq \text{Ps}_2e^-$.

Now, consider the actual statistics of particles in the bi-positronium ion Ps_2e^- . By applying ‘atomic analogies’ mentioned above one easily finds that the ground state in this system is the doublet state with the two independent spin functions

$$\chi_1 = (\alpha_e\beta_e\alpha_e - \beta_e\alpha_e\alpha_e)(\alpha_p\beta_p - \beta_p\alpha_p) \quad (13)$$

$$\chi_2 = (2\alpha_e\alpha_e\beta_e - \beta_e\alpha_e\alpha_e - \alpha_e\beta_e\alpha_e)(\alpha_p\beta_p - \beta_p\alpha_p) \quad (14)$$

where the indexes e and p designate electrons and positrons, respectively, while the notations α and β mean the one-electron spin-up and spin-down functions, respectively. The corresponding (spatial) projectors for the spatial parts of these two wave functions can be constructed explicitly, e.g., by using our method developed in [10]. The result of bound state computations with such trial wave functions is negative, i.e. the ground state in the Ps_2e^- ion is not bound. In fact, in this approach the Ps^- (and Ps^+) ion and bi-positronium Ps_2 are the only stable polyelectrons in which the total number of particles exceeds two.

Another approach to determine the statistics of particles in the bi-positronium ion Ps_2e^- follows from numerical calculations of the Ps_2 system. It was noticed in numerous calculation that one can rapidly decrease the variational energy of the Ps_2 system by introducing some special ‘exchange’ symmetry between positrons and electrons. In some works such a symmetry was called by the charge-conjugation symmetry. If four particles in the Ps_2 system are designated by numbers 1 (e^-), 2 (e^-), 3 (e^+) and 4 (e^+), then the total wave function of the ground (singlet) 1S –state can be written in the form

$$\Psi(\text{Ps}_2) = \frac{1}{8}(1 + \mathcal{P}_{13})(1 + \mathcal{P}_{12})(1 + \mathcal{P}_{34})\phi(1, 2, 3, 4) = \frac{1}{2}(1 + \mathcal{P}_{13})\Phi(1, 2, 3, 4) \quad (15)$$

where $\phi(1, 2, 3, 4)$ is the non-symmetrized wave function of the four-particles, while $\Phi(1, 2, 3, 4)$ is the wave function of the Ps_2 system symmetrized in respect with ‘atomic analogies’. The wave function $\Psi(\text{Ps}_2)$ has an additional symmetry in comparison with the $\Phi(1, 2, 3, 4)$ function and it allows one to produce very low total energies for the Ps_2 system.

Based on these results for the Ps_2 system we can split the bi-positronium ion Ps_2e^- into two systems: the central cluster Ps_2 and one electron e^- which moves in the field of this cluster. The cluster Ps_2 is a charge-conjugate system with the wave function symmetrized according in respect with Eq.(15). Now, all particles in the Ps_2 system become absolutely identical, while the outer-most electron is a particle with different permutation symmetry. The result of bound state computations with trial wave functions symmetrized in respect with this picture indicate clearly that the ground state in the Ps_2e^- ion is bound (in fact, it is well bound [11]). Moreover, it can be shown that in this approach the ground state in the six-particle polyelectron $\text{Ps}_2e^-e^+$ (or Ps_3) is also bound. In fact, if the total wave functions are symmetrized in respect to this approach, than many other polyelectrons are also bound. Furthermore, the total and binding energies of the bound (ground) states in all bound polyelectrons show smooth dependence upon the number of particles (or number of ‘electrons’). Below, such an approach to symmetrization of polyelectrons is called the united statistics.

The Ps_2e^- ion is only one example of systems for which the united statistics produces results which contradict statistics based on ‘atomic analogies’. Formally, this united statistics must be applied for all polyelectrons. For some polyelectrons, e.g., for Ps^- and Ps_2 , such new statistics leads to the same permutation symmetries of the total wave functions which is already known from ‘atomic analogies’. However, in higher polyelectrons, e.g., in Ps_2e^- , Ps_3 , etc, the united statistics produces the wave functions of the unusual symmetry (from the atomic point of view). In particular, if electrons and positrons in polyelectrons are allowed to form some very stable bosonic clusters (which are invariant in respect to charge conjugation), e.g., bi-positronium Ps_2 , then this will change the overall interparticle statistics (for more details, see discussion in [11]).

IV. PRODUCTION OF THE ELECTRON-POSITRON PAIRS AT HIGH-TEMPERATURE EXPLOSIONS.

In general, an intense creation of electron-positron pairs by high-energy photons (or γ -quanta) around every physical ‘body’ heated to extremely high temperatures, e.g., $T \geq 5 - 10 \text{ MeV}$, can be considered as an instability of the real media for propagating photons of high and very high energies. The total numbers of the newly created electrons N_- and

positrons N_+ rapidly increase with the temperature T and can be evaluated with the use of the following formula (see, e.g., [9])

$$N_+ \approx N_- \approx \frac{3\zeta(3)}{2\pi^2} \cdot V \cdot \left(\frac{kT}{\hbar c}\right)^3 \quad (16)$$

where V is the volume in which positrons are created and $\zeta(x)$ is the Riemann ζ -function [12] ($\zeta(3) \approx 1.202056903159594$). In fact, this number is small (and even very small) for $kT \ll m_e c^2$, but for $kT \approx m_e c^2$ the total number of positrons N_+ in $\approx 10^6 - 10^7$ times exceeds the numbers of incident atomic electrons located in the same volume V . At temperatures $kT \approx m_e c^2$ each newly created positron occupies the volume $V_0 \approx \lambda_e = \alpha a_0$, where $\lambda_e = \frac{\hbar}{m_e c} \approx 3.862 \cdot 10^{-13} m$ is the electron Compton wavelength, α is the fine structure constant and a_0 is the Bohr radius. This means that inside of the volume of U atom one finds almost $8 \cdot (\alpha)^{-3} \approx 2.057 \cdot 10^7$ positrons (and electrons) created due to the instability of the electromagnetic vacuum. Note that the total number of atomic electrons in the neutral U atom equals 92. This means that at such high temperatures, i.e. for $T \geq 511 keV$, we can neglect by all original atomic electrons and evaluate the total number of electrons by the same formula, Eq.(16).

Let us suppose that somehow we could create an overheated spatial area with the radius $\approx 1 m$. If the temperature of that area equals $T = \frac{m_e c^2}{k} \approx 5.93 \cdot 10^9 K$, then this overheated ball contains $\approx 7.272 \cdot 10^{37}$ electrons and positrons. The sudden annihilation of these (e^-, e^+) -pairs produces $\approx 1.434 \cdot 10^{38}$ γ -quanta. The total energy released during this annihilation explosion equals to thermal explosion of $\approx 1.564 \cdot 10^{12}$ tonnes of TNT, or $1.564 \cdot 10^6$ megatonnes of TNT. Such an energy significantly exceeds thermal energy released during any nuclear and/or thermonuclear explosion. If the radius of instability area is $1 cm$, then annihilation of all electron-positron pairs created in that area produces thermal energy which is equal to the energy released from an explosion of 1.564 megatonnes of TNT. It can be a very interesting direction to design various nuclear and thermonuclear explosive devices in which some macroscopic spatial areas are heated to very large temperatures, e.g., $T \approx 250 - 300 keV$ and higher.

Creation and following annihilation of large numbers of electron-positron pairs (or positrons, for short) during nuclear and thermonuclear explosions has been noticed since the middle of 1950's. The approximate evaluations made in that time indicated that $\approx 5\%$ of the total energy of a typical nuclear explosion is released in the form of newly cre-

ated electron-positron pairs. In modern multi-shell nuclear explosive devices with very light thermonuclear amplifiers the overall and local temperatures can be much higher. Therefore, the total numbers of positrons (and electrons) created during such explosions are also much higher. In general, the local numbers of created positrons/electrons (dN_+ and/or dN_-) directly depend upon the local temperature T (in eV) and volume dV of the spatial area. The exact formula takes the form

$$dN_+ = dN_- = \frac{dV}{\pi^2 \hbar^3} \int_0^\infty \frac{p^2 dp}{\exp\left(\frac{E}{T}\right) + 1} \quad (17)$$

where $E = c\sqrt{p^2 + m^2 c^2}$ is the relativistic energy of a particle with momentum p . This expression must be also integrated over the spatial volume of the high-temperature area inside of the nuclear explosive device.

Another direction which can be used to reach extremely high temperatures is based on the use of thermonuclear explosions. For instance, the temperature developed during a thermonuclear explosion of modern 550 kt (of TNT) warhead with the total mass 280 kg can be evaluated as

$$T \approx \frac{550,000,000,000,000 \cdot 8.617343 \cdot 10^{-5}}{280,000} \approx 169.3 \cdot 10^3 eV = 169.3 \text{ keV} \quad (18)$$

where we have assumed that 1 gram of TNT releases the energy 1000 *cal* during its explosion (see, e.g., [13]). The temperature T which is determined by Eq.(18) corresponds to the case when complete thermal equilibrium is established between remains of the warhead and radiation. The local temperatures achieved inside of this explosive device can be significantly higher ($T \approx 300 \text{ keV}$). Such temperatures are comparable with the corresponding threshold energy $\approx 511 \text{ keV}$ which is needed for creation of one positron e^+ . In general, for $T < 511 \text{ keV}$ the intensity of annihilation I_A in the electron-positron plasma per unit volume is

$$I_A = 4\pi\alpha^4 \left(\frac{c}{a_0}\right) \left(\frac{mT}{2\pi\hbar^2}\right)^3 \exp\left(-\frac{2m_e c^2}{T}\right) \quad (19)$$

where T is the temperature expressed in eV . For $T \approx 511 \text{ keV}$ the factor $\left(\frac{mT}{2\pi\hbar^2}\right)^3 \approx \frac{1}{8\pi^3} \left(\frac{mc}{\hbar}\right)^6 \sim \alpha^{-6}$, i.e. it is a very large number. The factor I_A defined in Eq.(19) is the intensity of electron-positron annihilations observed in one atomic volume $V_a = \frac{4\pi}{3}(a_0)^3$.

For regular nuclear and thermonuclear explosions we always have $T \ll 511 \text{ keV}$. In such cases the densities of the electrons n_- and positrons n_+ at equilibrium can be evaluated

from the formulas (see, e.g., [9])

$$n_- = \frac{n_0}{2} + \left[\left(\frac{n_0}{2} \right)^2 + \frac{1}{2} \left(\frac{mT}{2\pi\hbar^2} \right)^3 \exp\left(-\frac{2m_e c^2}{T}\right) \right]^{\frac{1}{2}} \quad (20)$$

$$n_+ = -\frac{n_0}{2} + \left[\left(\frac{n_0}{2} \right)^2 + \frac{1}{2} \left(\frac{mT}{2\pi\hbar^2} \right)^3 \exp\left(-\frac{2m_e c^2}{T}\right) \right]^{\frac{1}{2}} \quad (21)$$

where n_0 is the incident electron density. The formulas, Eqs.(19) - (21), can be applied to evaluate the actual positron/electron density and determine the intensity of annihilation I_A in the electron-positron plasma which forms during any nuclear and/or thermonuclear explosion. Our analysis indicates clearly that formation and annihilation of very large numbers of electron-positron pairs inside of modern nuclear and thermonuclear explosive devices is an important part of the explosion. Briefly, we can conclude that macroscopic polyelectrons extensively form during high-temperature thermal explosions, e.g., during any nuclear and/or thermonuclear explosion. The following annihilation of large numbers of electron-positron pairs can change the observed spectrum of soft γ -radiation emitted in such explosions.

In applications to astrophysics it is important to note that the area in which electromagnetic vacuum is unstable arises around each overheated physical body, e.g., around any star with very high temperature at its surface. Very large numbers of electrons and positrons are constantly created in such areas. In addition to thermal instability of electromagnetic vacuum there are many other ways in which positrons (and electrons) are also created. Later all these newly created particles begin to propagate into outer space and annihilate during their propagation. In general, annihilation of electron-positron pairs proceeds very intensively in spatial areas closed to the overheated body. At larger distances the overall intensity of the annihilation rapidly decreases with the distance. In general, these two spatially separated areas (area of vacuum instability and annihilation area) can be found around any overheated physical body. Moreover, the spatial radii of the both areas (instability and annihilation areas) rapidly increase with the temperature of the overheated body.

V. THERMAL SUPPORT OF THERMONUCLEAR BURN-UP.

Let us describe the problem in which thermal energy released from electron-positron annihilation can be used to simplify thermonuclear ignition. For simplicity, consider thermonuclear ignition of the compressed $\rho \geq 10 \text{ g} \cdot \text{cm}^{-3}$ equimolar deuterium-tritium mixture (below, DT-mixture, for short). As follows from numerous experiments and various theo-

retical evaluations after ignition in one spatial point (also called the central point) such a mixture burns very effectively by itself (see, e.g. [14]). Practically in all actual cases the burning wave propagates from the hot center as a high-temperature detonation wave. In reality, the compressed equimolar DT-mixture can be ignited by an incident nuclear charge either directly, i.e. by a propagating heat wave, or by using the phenomenon of radiative ablation. In the last case the ablating radiation flux comes from an incident nuclear explosion. The principal question is: can we ignite a highly compressed equimolar DT mixture without any preliminary nuclear explosion? In general, to achieve thermonuclear ignition one needs to compress the equimolar DT mixture to very large densities $\rho \geq 500 \text{ g} \cdot \text{cm}^{-3}$. If such a highly compressed equimolar DT mixture is confined, e.g., by a convergent consequence of shock waves, for some time (e.g., $\tau \geq 1 \cdot 10^{-9} \text{ sec}$), then it burns up by itself. In actual systems such a confined time must be even longer, since it is very hard to exclude all possible energy losses.

The current answer to the question formulated above is negative, since at this time it is impossible to reach such high densities and relatively long confinement times for the equimolar DT mixture by using only chemical explosives. This means that the shock wave which propagates from the hot center of highly compressed equimolar DT-mixture has temperatures which are not sufficient for thermonuclear ignition of this mixture. This situation can be changed, in principle, by saturating the compressed DT-mixture with positrons. The following annihilation of electron-positron pairs and absorption of annihilation γ -quanta in the dense DT-mixture can be used as an additional thermal source which can amplify and accelerate the shock wave propagating from hot center. The amplified and accelerated shock wave can produce thermonuclear ignition.

The governing equation takes the form (see, e.g., [15])

$$\mathcal{C} \cdot \frac{dT}{dt} = -\mathcal{C} \cdot \frac{3}{r_f} \cdot \frac{dr_f}{dt} \cdot T + \tilde{q}(r_f, T, \rho_0) = -\mathcal{C} \cdot \frac{3}{r_f} \cdot V_{\max} \cdot T + \tilde{q}(r_f, T, \rho_0) \quad (22)$$

where T is the temperature in the hot zone and \mathcal{C} is the specific heat (in $MJ \cdot g^{-1} \cdot keV^{-1}$, where $1MJ = 1 \cdot 10^6 J$) per unit mass of thermonuclear fuel. In the first approximation the specific heat \mathcal{C} does not depend upon T . In fact, such an approximation has a very good accuracy if: (1) $T \geq 0.1 \text{ keV}$, and (2) the thermonuclear fuels contains only light elements (with $A \leq 20$ and $Z \leq 10$). In the last equation $\tilde{q}(r_f, T, \rho_0)$ is the so-called energy release function (per unit volume).

In general, Eq.(22), represents the burn-up problem in its classical form [15]. Let $r_f(t) \geq r_0$ be the radius of the hot, spherical spot created by the shock (thermal) wave to the time t . The velocity of the hot zone expansion is $\frac{dr_f}{dt}$, respectively. In reality, the hot zone expands either by the high-temperature thermal wave $V_t = (\frac{dr_f}{dt})_t$, or by the detonation wave $V_d = (\frac{dr_f}{dt})_d$. The actual (or observed) velocity of the hot zone expansion V_{\max} is largest of the two corresponding velocities, i.e. $V_{\max} = \max\left[(\frac{dr_f}{dt})_d, (\frac{dr_f}{dt})_t\right]$. The temperature behind the shock (or thermal) wave $T = T_f$ is significantly larger than the initial temperature before such a wave, where $T_0 \approx 0$. Now, by introducing the burn-up parameter $x = \rho_0 \cdot r_f$, one can re-write the burn-up equation, Eq.(22), to the the form (see [15]):

$$\frac{dT}{dx} = -\frac{3}{x} \cdot T + \frac{q(x, T, \rho_0)}{\mathcal{C} \cdot V_{\max}} \quad (23)$$

where $q(x, T, \rho_0) = \frac{\tilde{q}(r_f, T, \rho_0)}{\rho_0}$ is the so-called normalized energy release function (per unit mass). In contrast with r_f , the burn-up parameter x does not depend significantly on ρ_0 . The explicit expressions for the $q(x, T, \rho_0)$ function and for the \mathcal{C} and V_{\max} values depend significantly on the ionic contents of the considered thermonuclear fuel. For equimolar DT-mixture the explicit form of the $q(x, T, \rho_0)$ function is very well known (see, e.g., [5], [15]).

Numerical solution of the burn-up equation, Eq.(23), is straightforward, if the function $q(x, T, \rho_0)$ is known. For given density ρ_0 and temperature T this equation allows one to determine the minimal value of the burn-up parameter x_{cr} . Solution of Eq.(23) for different temperatures produces the explicit dependence of minimal (or critical) burn-up parameter upon the temperature T . In other words, for any given density ρ of the equimolar DT-mixture we obtain the burn-up curve $x_{cr}(T)$. As follows from the results of earlier works (see, e.g., [14], [15]) the minimal burn-up parameter rapidly decreases with the temperature. Briefly, this means that some under-critical equimolar DT-mixture can be transformed into over-critical DT-plasma (with the same density) by increasing the central temperature. Here the over-critical DT-plasma means the high-temperature equimolar DT-plasma in which thermonuclear burn-up is possible. The idea to use an additional thermal support for thermonuclear burn-up is based on this fact. If the equimolar DT-mixture is saturated by positrons, then the thermal energy released from electron-positron annihilation can be used to simplify thermonuclear ignition. This approach can be used in some applications, including modern experiments which are based on laser-driven ablative implosion scheme to compressed small DT-microspheres.

VI. CONCLUSIONS.

We have considered the phenomena of annihilation explosions and photon detonation in macroscopic polyelectrons. In general, the macroscopic polyelectrons can be created experimentally with the use of radiative ablation. Theoretical and experimental study of such polyelectrons is of great interest in a number of applications. In general, the macroscopic properties of polyelectrons are different from analogous properties of regular (i.e. atomic) systems. A very fast compression of macroscopic polyelectrons creates conditions at which almost sudden annihilation of very large number of electron-positron pairs is possible. This phenomenon is called the photon explosion. It can be used to develop various explosive devices based on electron-positron annihilation. Moreover, in the dense electron-positron plasma, annihilation of particles can propagate as a spatial surface which, in fact, coincides with the detonation wave. In such cases we deal with the photon detonation. Currently, there are a number of experimental restrictions which are crucial for workability of devices based on photon explosions. In the future we might expect that the experimental situation with macroscopic polyelectrons will be improved.

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